# Electronic Structure of $\mathrm{LaF}^{+}$and LaF from Frozen-Core Four-Component Relativistic Multiconfigurational Quasidegenerate Perturbation Theory 

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#### Abstract

The electronic structure of the molecules $\mathrm{LaF}^{+}$and LaF was studied using frozen-core four-component multiconfigurational quasidegenerate perturbation theory. To obtain proper excitation energies for $\mathrm{LaF}^{+}$, it was essential to include electronic correlations between the outermost valence electrons ( $4 \mathrm{f}, 5 \mathrm{~d}$, and 6 s ) and ionic core electrons composed of $(4 \mathrm{~s}, 4 \mathrm{p}, 4 \mathrm{~d}, 5 \mathrm{~s}$, and 5 p$)$. The lowest-lying 16 excited states were examined for $\mathrm{LaF}^{+}$, and the lowest 30 states were examined for LaF . The excitation energies calculated for $\mathrm{LaF}^{+}$agree with the available experimental values, as well as with values from ligand field theory. Errors are within 0.4 eV ; for example, the highest observed state ${ }^{2} \Pi$ is 3.77 eV above the ground state, and the present value is 4.09 eV . For LaF, agreement between the experimental and theoretical state assignments and between the experimental and calculated excitation energies was generally good, except for the electron configurations of certain states. Errors are within 0.4 eV except for a single anomaly; for example, the highest observed excitedstate discussed in this work is 2.80 eV above the ground state, and the present value is 2.42 eV . We discuss the characteristics of the bonding in $\mathrm{LaF}^{+}$and LaF .


## 1. Introduction

$\mathrm{LaF}^{+}$and LaF can be studied either as a transition metal fluoride or as a lanthanide metal fluoride. Experimentally, Shenyavskaya and Gurvich ${ }^{1}$ found the ground state of $\mathrm{LaF}^{+}$to be $(6 s)^{1}{ }^{2} \sum$. Kaledin, Kaledin, and Heaven ${ }^{2}$ then asserted that the ground state is $(5 \mathrm{~d})^{12} \Delta$ with $\Omega=1.5$, using laser adsorption spectra and ligand field theory (LFT). The excited states of this ion were studied extensively by these authors. Because $\mathrm{LaF}^{+}$ is the simplest lanthanide monofluoride, we investigate it in depth as a basis for discussing the electronic structure of the larger lanthanide compounds. We study it here using fourcomponent relativistic theory. It was necessary to include electronic correlations between the valence and ion cores, including the $4 \mathrm{~s}, 4 \mathrm{p}$, and 4 d electrons, to give the appropriate designations for the excited states; see Section 3.

The LaF visible band was first studied by Barrow, Bastin, and Moore. ${ }^{3}$ Schall, Linton, and Field ${ }^{4}$ identified the ground state as $(6 \mathrm{~s})^{21} \sum$. Since then, there have been many experimental studies of the excited states. ${ }^{5-11}$ (A review of spectroscopic properties of rare earths is given in ref 12.) Only three theoretical works have been published for LaF, however. Schall, Dulick, and Field ${ }^{13}$ discussed the excited states of LaF using LFT. Hong, Dolg, and $\mathrm{Li}^{14}$ gave the spectroscopic constants for lanthanide monoxides and monofluorides including LaF, using the scalarrelativistic zeroth-order regular approximation and Douglas-Kroll-Hess approximation. Fahs and co-workers ${ }^{15}$ investigated the excited states of LaF. In their calculation, 46 core electrons in La are treated via pseudopotentials, and the actual number of electrons is $20\left(=\mathrm{La} 5 \mathrm{~s}^{2} 5 \mathrm{p}^{6} 5 \mathrm{~d}^{1} 6 \mathrm{~s}^{2}+\mathrm{F} 1 \mathrm{~s}^{2} 2 \mathrm{~s}^{2} 2 \mathrm{p}^{5}\right)$. Of these,

[^0]8 electrons (La $5 \mathrm{~d}^{1} 6 \mathrm{~s}^{2}+\mathrm{F} 2 \mathrm{p}^{5}$ ) were selected as active, and the remaining 12 were frozen. Fahs and co-workers ${ }^{15}$ performed complete-active-space self-consistent-field (CASSCF) calculations with 12 active molecular orbitals assembled from s and d atomic orbitals. After CASSCF, multireference configuration interaction (CI) calculations were performed to account for correlation effects. Spin-orbit effects were introduced semiempirically. ${ }^{15}$ Configurations including the 6 p spinors are disregarded in their calculations. Although the resulting excitation energies generally agree with experiment, the present work will demonstrate that the contributions of the La 6 p spinors are significant in some excited states assigned as ( $5 \mathrm{~d}^{1} 6 \mathrm{~s}^{1}$ ).
We know of no investigations using four-component relativistic theory. We therefore studied LaF using the fourcomponent relativistic theory, to establish the designations for the excited states. The calculated excitation energies generally agree with those of experiment but in some states contradict the experimental electronic configuration. ${ }^{4}$
We also discuss the characteristics of the chemical bond of $\mathrm{LaF}^{+}$and LaF . For example, LaF is considered to be the ionic compound $\mathrm{La}^{+} \mathrm{F}^{-}$. In the atomic ion, the electronic configuration of the $\mathrm{La}^{+}$ground state is $(5 \mathrm{~d})^{2}$, but the electronic configuration of the ground state of LaF is $(6 \mathrm{~s})^{2}$-like. We shall explain this.

Section 2 sets out the method of the calculations. Section 3 then discusses the electronic structure of $\mathrm{LaF}^{+}$and LaF . Section 4 offers concluding remarks.

## 2. Method of the Calculation

2.1. RFCA and Basis Set. Because it is difficult to treat all 57 of the electrons of the La atom, we used the reduced frozencore approximation (RFCA) proposed by Matsuoka and Watanabe. ${ }^{16,17}$ We prepared four ionic cores for the La atom, because in the preliminary calculations the errors given by the
large core, ordinarily accepted as appropriate (see C1 and C2 below), lead to large errors in the calculated excitation energies. We categorize electrons into three sets: frozen-core (f-core) electrons; active-core (a-core) electrons from which one and two electron excitations are allowed but are not treated as valence electrons in CASCI; and the valence electrons (val.)

C1, f-core $\{\mathrm{Cd}(48)+\mathrm{He}(2)\}+\mathrm{a}$-core $\left\{\left(5 \mathrm{p}^{6}\right)+\left(2 \mathrm{~s}^{2} 2 \mathrm{p}^{6}\right)\right\}+$ val.;

C2, f-core $\{\operatorname{Pd}(46)+$ He (2) $\}+$ a-core $\left\{\left(5 s^{2} 5 p^{6}\right)+\left(2 s^{2} 2 p^{6}\right)\right\}$ + val.;

C3, f-core $\{\mathrm{Kr}(36)+\mathrm{He}(2)\}+\mathrm{a}$-core $\left\{\left(4 \mathrm{~d}^{10} 5 \mathrm{~s}^{2} 5 \mathrm{p}^{6}\right)+\left(2 \mathrm{~s}^{2}-\right.\right.$ $\left.\left.2 p^{6}\right)\right\}+$ val.;

C 4 , f-core $\left\{\mathrm{Zn}^{2+}(28)+\mathrm{He}(2)\right\}+$ a-core $\left\{\left(4 \mathrm{~s}^{2} 4 \mathrm{p}^{6} 4 \mathrm{~d}^{10} 5 \mathrm{~s}^{2}-\right.\right.$ $\left.\left.5 p^{6}\right)+\left(2 s^{2} 2 p^{6}\right)\right\}+$ val.;

As an example, the notation "f-core $\{\mathrm{Cd}(48)+\mathrm{He}(2)\}$ " indicates that the frozen-core is composed of a Cd -like ion core in La, having 48 electrons, and a He -like ion core in F , having 2 electrons that are fixed to the atomic spinors. Also "a-core $\left\{\left(5 p^{6}\right)+\left(2 s^{2} 2 p^{6}\right)\right\} "$ indicates that the active-core is composed of $\mathrm{La}\left(5 p^{6}\right)$ and $\mathrm{F}\left(2 \mathrm{~s}^{2} 2 \mathrm{p}^{6}\right)$. The number of active electrons for $\mathrm{C} 1-\mathrm{C} 4$ is $14-34$. The number of valence electrons is 1 or 2 depending on whether $\mathrm{LaF}^{+}$or LaF is treated. The basis set for the respective calculations of $\mathrm{C} 1-\mathrm{C} 4$ are rather large. For example, the set for C 4 is $\mathrm{La}[1 * 6 / 1 * 5+(11) / 1 * 7 / 1 * 8 /(1)]+$ $\mathrm{F}[21 / 422 /(1)]$, where a slash separates the symmetries $\mathrm{s}, \mathrm{p}, \mathrm{d}$, f , and g symmetries, $1^{*} n$ implies that $n$ primitive Gaussian type functions (pGTFs) are used, and a 2 and 4 indicate that the contracted GTFs (cGTF) are spanned with 2 and 4 primitives, respectively. The (11) and (1) for La are two p-type polarization functions ${ }^{18}$ and one g-type polarization function, whereas (1) for F is a single d-type polarization function. ${ }^{18}$ The total number of molecular spinors generated is 145 .

The La pGTFs in the parentheses are those of the most diffuse GTFs in the respective atomic spinors given by Koga, Tatewaki, and Matsuoka (KTM). ${ }^{19}$ The 8 f-type primitives are generated in the present work. The La p-type polarization functions have a similar diffuseness (exponents are $0.041,0.012$ ) to that of the s-type pGTFs for the 6 s atomic spinor (exponents are 0.055 , 0.023 ), so that we have not added further p primitives. For the F a-core, cGTFs are constructed from the atomic spinors given by KTM. ${ }^{20}$
2.2. Complete Active Space Configuration Interaction (CASCI) and Multiconfigurational Quasidegenerate Perturbation Theory (MC-QDPT). 2.2.1. $\mathrm{LaF}^{+}$. For $\mathrm{LaF}^{+}$, we first performed RFCA Dirac-Fock-Roothaan (DFR) calculations for $\mathrm{LaF}^{2+}$. $\left\{\mathrm{La}^{3+}\left(5 \mathrm{p}^{6}\right) \mathrm{F}^{-}\left(2 \mathrm{p}^{6}\right)\right\}^{2+}$ provides appropriate valence spinors for the $\mathrm{LaF}^{+}$ground and excited states, having the electronic configurations $(\mathrm{LaF})^{2+} n l^{1}$, because $\mathrm{LaF}^{2+}$ gives the electronic field generated by the $m(m=14-34)$ activecore electrons, which equivalently acts on the virtual spinors in which a $n l$ electron moves. In the next section, we shall see the adequateness of the resulting spinors for $\mathrm{LaF}^{+}$. Second, assuming the no virtual pair approximation, ${ }^{21-26}$ we performed CASCI ${ }^{27,28}$ calculations using $\mathrm{LaF}^{2+}$ virtual (valence) spinors, filling one electron in the respective spinors. Third, to account for electron correlation effects among the valence electrons and between the valence and active-core electrons, we performed MC-QDPT ${ }^{28}$ calculations. The matrix element of MC-QDPT is expressed as

$$
\begin{align*}
H_{\mu \nu}= & E_{\mu}^{G C S-C I} \delta_{\mu \mathrm{v}}+ \\
& \frac{1}{2} \sum_{I \notin G C S}\left\{\frac{\langle\mu| H_{D C}^{+}|I><I| H_{D C}^{+} \mid v>}{E_{v}^{(0)}-E_{I}^{(0)}}+H . C .\right\} \tag{1}
\end{align*}
$$

where $\mu$ and $v$ denote CASCI eigenfunctions. The one and two electron excitations from the active-core and valence shells to all the valence and virtual spinors (denoted as $I$ ) were taken into account where the CSFs included in the CASCI are excluded. ${ }^{28}$ We did not encounter intruder states ${ }^{29-31}$ in this work. The appropriateness of the core $\mathrm{C} 1-\mathrm{C} 4$ is tested, calculating the excitation energies. It emerges that the smallest core, C4, is needed. Moreover, CAS space spanned with the lowest 30 virtual spinors of $\mathrm{LaF}^{2+}$ was found to be necessary to obtain reasonable excitation energies.
2.2.2. LaF. For LaF, using the C 4 model, we performed RFCA DFR calculations for $\mathrm{LaF}^{+}$, then performed CASCI, filling 2 electrons in the lowest 30 virtual (valence) molecular spinors. Because $\mathrm{Zn}^{2+}+\mathrm{He}$ cores are used, it is taken into account the correlation effects between the 2 valence and 34 active-core electrons in MC-QDPT. We scarcely met intruder states ${ }^{29-31}$ as in $\mathrm{LaF}^{+}$.

The present MC-QDPT program treats distinctly the two strings of subspecies due to time reversal symmetry; for example, we actually used 60 valence spinors rather than 30 spinors.
2.3. Spectroscopic Constants. Using the potential curves given by MC-QDPT, we obtained the spectroscopic constants of $R_{\mathrm{e}}$ and $\omega_{\mathrm{e}}$ and excitations of $T_{\mathrm{e}}$ (electronic transition energies between the two potential minima) and $T_{0}$ (electronic transition energies between the corresponding 0 vibrational energies) by solving the one-dimensional Schrödinger equation using the Numerov method; $;{ }^{32,33}$ the potential curves are fitted to the fifthdegree polynomial of internuclear distance $R$, and $\omega_{\mathrm{e}}, \omega_{\mathrm{e}} x_{\mathrm{e}}$, and $\omega_{\mathrm{e}} y_{\mathrm{e}}$ are determined using the lowest three vibrational states of the respective symmetries. We compiled only $\omega_{\mathrm{e}}$ in this work.

## 3. Results

3.1. $\mathrm{LaF}^{+}$. 3.1.1. $D F R$ Calculation. Because it is considered that the $\mathrm{LaF}^{2+}$ ion provides appropriate molecular spinors for various $\mathrm{LaF}^{+}$electronic states including the ground state, we first performed DFR calculations for the $\mathrm{LaF}^{2+}$ ion, using the four La ion core models listed in Section 2.1. Although we have not shown detailed results with these cores except for C4, the respective core models gave similar energetics so far as DFR is concerned; the total energies are similar, irrespective of the ion core size, and the same is true for the occupied and virtual spinors. For example, the DFR total energies for $\mathrm{LaF}^{2+}$ given at $R=3.75$ bohrs ( $1.984 \AA$ ) by C1, C2, C3, and C4 are $-8592.7503,-8592.7525,-8592.7522$, and -8592.7507 hartrees, respectively, and the highest occupied spinors having characteristics of $\mathrm{F}\left(2 \mathrm{p}_{+}\right)_{1 / 2}$ are $-1.0054,-1.0066,-1.0068$, and -1.0070 hartrees, respectively. The spinor energies and gross atomic orbital populations for C 4 are shown in Table 1, and the contour maps for the important spinors are displayed in Figure 1. The total electron number in La is 26.5, and for F is 7.5. Formally, atomic $\mathrm{La}^{2+}$ takes the electronic configuration $\left(4 s^{2} 4 p^{6} 4 d^{10} 5 s^{2} 5 p^{6} 5 d^{1}\right)$, including 27 electrons, and $F$ is $\left(2 s^{2}-\right.$ $2 p^{5}$ ) including 7 electrons. From Table 1, we see in $\mathrm{LaF}^{2+}$ that 0.5 La 5 d electrons move into F 2 p spinors to form $\left\{\mathrm{La}^{2.5+}{ }^{-}\right.$ $\left.\left(5 \mathrm{p}^{6} \mathrm{~d} * 0.4 \mathrm{f} * 0.1\right) \mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$, where $\mathrm{d}^{*}$ and $\mathrm{f}^{*}$ are the polarization functions of La that form molecular spinors with F 2ps. A single valence electron in $\mathrm{LaF}^{+}$thus moves in the field generated by $\left\{\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d} *{ }^{0.4} \mathrm{f}^{* 0.1}\right) \mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$.

We may find a similar charge transfer in $\mathrm{LaF}^{+} \mathrm{DFR}$, suggesting that the chemical bonding in LaF arises from charge transfer from the $\mathrm{La}^{2+}$ ion to the F atom (see Section 3.3).
3.1.2. Verification of the Appropriateness of the Ion Core. When performing CASCI, we should determine the number of

TABLE 1: Spinor Energies (hartrees) and GAOPS of LaF ${ }^{2+}$ at $R=2.002 \AA$ with C4 $4^{a, b}$

${ }^{a}$ This is $R_{\mathrm{e}}$ of the $\mathrm{LaF}^{+}$ground state from C4 MC-QDPT; the DFR total energy for $\mathrm{LaF}^{2+}$ is -8592.749625 hartrees. ${ }^{b}$ The lower first to ninth spinors and the higher 31 st to 145 th spinors are not shown to save space.
valence spinors which the valence electron occupies. In the Dirac-Fock procedure, any spinors with the same $\Omega$ mix with each other; in many cases, we cannot distinguish the spinors purely as $\mathrm{s}, \mathrm{p}, \mathrm{d}$, and f atomic spinors. The $\mathrm{s}, \mathrm{p}, \mathrm{d}$, and f atomic spinors yield $7,5,3$, and 1 molecular spinors for the $\Omega=1 / 2$, $3 / 2,5 / 2$, and $7 / 2$ symmetries, respectively. If we select the 16 lowest virtual spinors in $\left\{\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d} * 0.4 \mathrm{f} * 0.1\right) \mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$, then we may include all the necessary atomic spinors, but we select 19 spinors for safety; these are composed of $9,6,3$, and 1 spinors for the $\Omega=1 / 2,3 / 2,5 / 2$, and $7 / 2$ symmetries.

We then performed MC-QDPT calculations using the C1C 4 core models. The number of valence electrons in $\mathrm{LaF}^{+}$is 1 , and we recall the numbers in the active-core for $\mathrm{C} 1-\mathrm{C} 4$ is $14-$ 34. The electronic excitation energies ( $T_{0}$ ) with MC-QDPT are given in columns $3-6$ of Table 2 and can be compared with the experimental values in the final column. Irrespective of the core models, the calculated ground state is always $(5 d)_{3 / 2}{ }^{1}$, similar to atomic $\mathrm{La}^{2+}$ (see the 18th spinor in Table 1 and Figure 1 ), which is consistent with experiment; the subscript $3 / 2$ and the superscript 1 , respectively, denote the total electronic angular momentum $\Omega$ of the spinor under consideration and the number of electrons in $(5 \mathrm{~d})_{3 / 2}$.

We note the excitation energies for $(4 \mathrm{f})_{5 / 2}{ }^{1}$ and $(4 \mathrm{f})_{7 / 2}{ }^{1}$ (see 27th and 28th spinors in Table 1 and Figure 1) because the electronic spectra including $f$ electrons have particular importance in lanthanide chemistry. The calculated excitation energies with C 1 are 3.32 and 3.53 eV , which are far from the experimental values of 2.05 and 2.12 eV , but they gradually approach the experimental values as the core varies from C 1 to C 4 . The best is given by C 4 , which is reasonable because the atomic 4 f spinors occupy the same special region as the atomic $4 d, 4 p$, and 4 s spinors (as shown in Figure 2), and all of them are included in C 4 . The calculated excitation energies for $(4 \mathrm{f})_{5 / 2}{ }^{1}$ and $(4 \mathrm{f})_{7 / 2}{ }^{1}$ with C 4 are 2.10 and 2.41 eV , closer to the experimental values of 2.05 and 2.12 eV . We believe that the calculated excitation energy for $(4 \mathrm{f})_{7 / 2}{ }^{1}$ is a little larger. We therefore expand the spinor space included in CASCI and used the lowest 30 virtual spinors (instead of 19 spinors) as the valence spinors for $\mathrm{LaF}^{+}$. The results are given in column 7 of

Table 2. The calculated excitation energy to $(4 \mathrm{f})_{7 / 2}{ }^{1}$ becomes 2.16 eV . Agreement between this C4 calculation and experiment is reasonable. There is similar agreement between this and LFT calculations; the semiempirical parameters in LFT may implicitly include correlation effects between the valence and $4 \mathrm{~s}, 4 \mathrm{p}$, and 4 d electrons as well as the 5 s and 5 p electrons.

We also found that the 19 virtual spinor sets give an unnaturally higher MC-QDPT solution in some states at smaller internuclear distances, but the 30 spinor set does not. Henceforth we use the C 4 model with the lowest 30 virtual spinors of $\mathrm{LaF}^{2+}$; the $\mathrm{LaF}^{2+} 145 \mathrm{DFR}$ spinors are consequently divided into the 17 active core, 30 valence, and 98 virtual spinors of $\mathrm{LaF}^{+}$.
3.1.3. Excited States and Their Spectroscopic Constants. Table 3 shows the assignment of the states, excitation energies, and spectroscopic constants by experiment, LFT, and C4 MCQDPT. In the assignments, the approximate gross atomic orbital populations (GAOPs) ${ }^{34}$ given by MC-QDPT are included in the form $(\lambda)^{\operatorname{GAOP} \lambda}(\lambda=\mathrm{s}, \mathrm{p}, \mathrm{d}$, and f$)$

$$
\begin{equation*}
\mathrm{GAOP}_{\lambda}=\sum_{I} \mathrm{C}_{I}^{2} \mathrm{GAOP}_{I \lambda} \tag{2}
\end{equation*}
$$

where $\lambda, I$, and $C_{I}$ respectively indicate the symmetry of the atomic spinor, configuration, and mixing configurational coefficient in MC-QDPT. The important configurations are also included in the assignment, where the numbers in the Slater determinant $|\ldots|$ are spinors given in Table 1; the spinors for the active-core are not shown, and a number with an underline such as " $\underline{i}$ " indicates the Kramer's partner of the spinor " $i$ ". The weights of the main configurations for the zeroth-ninth states are always greater than 0.9 except for the fifth state, indicating that use of the virtual spinors of $\mathrm{LaF}^{2+}$ as the valence spinors of the ground and excited states of $\mathrm{LaF}^{+}$is adequate. Two excitation energies $T_{0}$ and $T_{\mathrm{e}}$ are listed. No experimental $T_{\mathrm{e}}$ values are known, but the calculated $T_{\mathrm{e}}$ values are quite close to the calculated $\mathrm{T}_{0}$ values, which are themselves close to the experimental values.

We now discuss the f-excited states. One of the (f) $)_{5 / 2}{ }^{1}$ (the sixth excited state) remains at 1.86 eV above the ground state,


Figure 1. Contour maps of densities of the valence spinors of the ground state $\mathrm{LaF}^{2+}$, together with spinor energies. The caption " $n$; pop ${ }_{1}\left(l_{ \pm}\right)_{\lambda / 2}$ $+\operatorname{pop}_{2}\left(l_{ \pm}^{\prime}\right)_{\lambda / 2} \epsilon$ " for the respective contour maps denotes the $n$th spinor having total electronic angular momentum $\lambda / 2$, contributed largely from the La $l_{ \pm}$and $l^{\prime} \pm$ spinors having electronic populations pop ${ }_{1}$ and pop $p_{2}$ and followed by the spinor energy in hartrees. The horizontal and vertical ( $z$ - and $x$-) axes are in bohrs; $z$ covers -10 to 10 bohrs, and $x$ covers 0 to +10 bohrs. The circles on the $z$-axis at $z=0.0$ and 3.783 bohrs ( $2.002 \AA$ ) indicate the La and F nuclei, respectively. The outermost values of the contour line are $0.0001 e$ bohrs $^{-3}$. The contour maps for La 5 p and F 2 p are the sum of those for (the 12th, the 13th, and the 14th) and for (the 15 th, the 16 th, and the 17 th) spinors in Table 1 . The value on a contour is twice that of the neighboring one outside it. The electron numbers inside the outermost line are between 0.956 and 1.001 , except for the La 5 p-like and F 2 p -like spinors for which the electron numbers are 3.000 and 3.001 , respectively. (For the former, contributions from La 5 p are 2.852 and from others are 0.148 ; for the latter, contributions from F 2 p are 2.659 and from others are 0.342 .)

TABLE 2: Excitation Energies T $_{0}$ for LaF $^{+}(\mathrm{eV})^{a}$

| assignment |  | C1 | C2 | C3 |  |  | res | hers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| spnrs in CAS-CI |  | 19 | 19 | 19 | 19 | 30 |  |  |
| exptl | prsnt | $\mathrm{Cd}+\mathrm{He}$ | $\mathrm{Pd}+\mathrm{He}$ | $\mathrm{Kr}+\mathrm{He}$ | $\mathrm{Zn}^{2+}+\mathrm{He}$ |  | $\mathrm{LFT}^{\text {b }}$ | exptl ${ }^{\text {c }}$ |
| ${ }^{2} \Delta_{3 / 2}$ | (d) $3 / 2{ }^{1}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| ${ }^{2} \Delta_{5 / 2}$ | (d) ${ }_{5 / 2}{ }^{1}$ | 0.143 | 0.145 | 0.149 | 0.147 | 0.151 | 0.174 | 0.034 |
| ${ }^{2} \Phi_{5 / 2}$ | (f) ${ }_{5 / 2}{ }^{1}$ | 3.315 | 3.070 | 2.544 | 2.097 | 1.862 | 1.996 | 2.050 |
| ${ }^{2} \Phi_{7 / 2}$ | (f) $7 / 2{ }^{1}$ | 3.529 | 3.301 | 2.826 | 2.408 | 2.159 | 2.170 | 2.120 |
| ${ }^{2} \Pi_{1 / 2}$ | (pf) 1/2 $^{1}$ | 3.477 | 3.312 | 4.037 | 3.965 | 4.001 | 3.596 | 3.749 |
| ${ }^{2} \Pi_{3 / 2}$ | (pf) ${ }_{3 / 2}{ }^{1}$ | 3.645 | 3.461 | 4.072 | 3.950 | 4.090 | 3.918 | 3.769 |

${ }^{a}$ Total energies for the ground state including zero-point vibrational energy are $-8593.550669,-8593.595713,-8594.157965$, and -8594.433519 hartrees for $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3$, and C 4 , respectively. ${ }^{b}$ LFT stands for ligand field theory. See ref 2 . ${ }^{c}$ See ref 2.
TABLE 3: Assignments of States, Excitation Energies, and Spectroscopic Constants of LaF ${ }^{+}$by C4 MC-QDPT ${ }^{a}$

|  | exptl asgn ${ }^{\text {b }}$ |  | prsnt asgn |  |  | $T_{0}(\mathrm{eV})$ |  |  | $T_{\mathrm{e}}(\mathrm{eV})$ | $R_{\text {e }}(\AA)$ |  | $\omega\left(\mathrm{cm}^{-1}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| no. | sym | config | sym | GAOP | important $\mathrm{CSF}^{c}$ | exptl ${ }^{\text {b }}$ | $\mathrm{LFT}^{\text {b }}$ | prsnt | prsnt | exptl ${ }^{\text {d }}$ | prsnt | exptl ${ }^{\text {d }}$ | prsnt |
| 0 | ${ }^{2} \Delta_{3 / 2}$ | d | 3/2 | (d) ${ }^{1}$ | 1.00\|18|+... | 0.000 | 0.000 | 0.000 | 0.000 | 1.998 | 2.002 | 603.6 | 521.6 |
| 1 | ${ }^{2} \Delta_{5 / 2}$ | d | 5/2 | (d) ${ }^{1}$ | $0.85\|20\|-0.52\|20\|+\ldots$ | 0.034 | 0.174 | 0.151 | 0.151 | 1.998 | 2.000 | 605.2 | 523.9 |
| 2 |  |  | 1/2 | (s) $)^{0.8}(d)^{0.1}$ | $0.94\|19\|-0.30\|\overline{19}\|+\ldots$ |  |  | 0.293 | 0.288 |  | 1.966 |  | 612.4 |
| 3 |  |  | 1/2 | (p) ${ }^{0.1}(\mathrm{~d})^{0.8}(\mathrm{f})^{0.1}$ | $0.70\|\overline{21}\|+0.66\|21\|+\ldots$ |  |  | 0.812 | 0.813 |  | 2.057 |  | 516.3 |
| 4 |  |  | 3/2 | (p) ${ }^{0.1}(\mathrm{~d})^{0.8}(\mathrm{f})^{0.1}$ | 0.96\|22|-0.22|33|+... |  |  | 0.902 | 0.903 |  | 2.056 |  | 516.2 |
| 5 |  |  | 1/2 | (s) $)^{0.1}(\mathrm{p})^{0.3}(\mathrm{~d})^{0.4}(\mathrm{f})^{0.2}$ | $0.71\|\underline{23}\|+0.41\|23\|+0.38\|\underline{26}\|+\ldots$ |  |  | 1.533 | 1.530 |  | 2.054 |  | 570.7 |
| 6 | ${ }^{2} \Phi_{5 / 2}$ | f | 5/2 | (f) ${ }^{1}$ | $0.80\|\overline{27}\|-0.54\|27\|+\ldots$ | 2.050 | 1.996 | 1.862 | 1.863 | 2.065 | 2.078 | 555.8 | 514.3 |
| 7 |  |  | 3/2 | (f) ${ }^{1}$ | $0.81\|\overline{29}\|+0.52\|29\|+\ldots$ |  |  | 1.995 | 1.996 |  | 2.070 |  | 502.6 |
| 8 | ${ }^{2} \Phi_{7 / 2}$ | f | 7/2 | (f) ${ }^{1}$ | $0.93\|\overline{28}\|+0.29\|28\|+\ldots$ | 2.120 | 2.170 | 2.159 | 2.159 | 2.065 | 2.080 |  | 516.4 |
| 9 |  |  | 5/2 | (f) ${ }^{1}$ | $0.71\|30\|-0.66 \mid 3 \overline{0 \mid}+\ldots$ |  |  | 2.234 | 2.236 |  | 2.070 |  | 504.5 |
| 10 |  |  | 1/2 | (s) ${ }^{0.3}(\mathrm{p})^{0.2}(\mathrm{~d})^{0.1}(\mathrm{f})^{0.3}$ | $0.59\|31\|-0.56\|\overline{32}\|+\ldots$ |  |  | 2.482 | 2.483 |  | 2.096 |  | 517.1 |
| 11 |  |  | 3/2 | (p) ${ }^{0.2}(\mathrm{~d})^{0.1}(\mathrm{f})^{0.7}$ | $0.86\|\overline{33}\|-0.39\|\overline{25}\|+\ldots$ |  |  | 2.572 | 2.574 |  | 2.083 |  | 507.4 |
| 12 |  |  | 1/2 | (s) $)^{0.1}(\mathrm{p})^{0.4}(\mathrm{~d})^{0.2}(\mathrm{f})^{0.3}$ | $0.57\|26\|-0.45\|23\|-0.37 \mid \underline{26 \mid}+\ldots$ |  |  | 3.169 | 3.172 |  | 2.106 |  | 471.4 |
| 13 | ${ }^{2} \Pi_{1 / 2}$ | p | 1/2 | (s) ${ }^{0.1}(\mathrm{p})^{0.7}(\mathrm{~d})^{0.1}(\mathrm{f})^{0.1}$ | $0.84\|24\|-0.32\|24\|+\ldots$ | 3.749 | 3.596 | 4.001 | 3.993 | 1.976 | 1.971 |  | 647.0 |
| 14 | ${ }^{2} \Pi_{3 / 2}$ | p | 3/2 | (p) ${ }^{0.8}(\mathrm{~d})^{0.1}(\mathrm{f})^{0.2}$ | $0.91\|25\|+0.39\|33\|+\ldots$ | 3.769 | 3.918 | 4.090 | 4.082 | 1.976 | 1.968 |  | 666.0 |
| 15 |  |  | 1/2 | (s) $)^{0.1}(\mathrm{p})^{0.4}(\mathrm{~d})^{0.1}(\mathrm{f})^{0.4}$ | $0.54\|34\|+0.52\|26\|+\ldots$ |  |  | 6.317 | 6.310 |  | 1.992 |  | 653.3 |
| 16 |  |  | 1/2 | $(\mathrm{s})^{0.3}(\mathrm{p})^{0.2}(\mathrm{~d})^{0.1}(f)^{0.4}$ | $0.61\|\underline{32}\|+0.56\|\underline{31}\|+\ldots$ |  |  | 6.923 | 6.907 |  | 1.976 |  | 789.4 |

${ }^{a}$ TEs with and without zero point vibrational energy are -8594.433519 and -8594.434718 hartrees, respectively. ${ }^{b}$ See ref $2 .{ }^{c}$ The numbers in a determinant $|\ldots|$ are the spinors given in Table 1 , and a spinor $(\underline{i})$ with an underbar indicates the Kramer's partner of $(i)$. ${ }^{d}$ See ref 1 .
and the other $(\mathrm{f})_{5 / 2}{ }^{1}$ (the ninth excited state) is at 2.23 eV above. Let us consider why this difference arises. The former state consists almost completely of $0.80|\ldots .27|-0.54|\ldots 27|$, and the latter $0.71|\ldots 30|-0.66|\ldots 30|$. From Figure 1, we see that the 30th spinor, $\left(4 \mathrm{f}_{+}\right)_{5 / 2}$-like, has a lobe where F 2 p has significant density, causing the 30th spinor energy to be higher than that of the $\left(4 f_{-}\right)_{5 / 2}$-like 27th spinor, and also the state with 30th to be energetically higher than 27 th.

The potential curves are shown in Figure 3. If the CSF, including diffuse contributors such as the 19th, 24th, and 25th spinors characterized by the 6 s and 6 p atomic spinors (see Figure 1), has significant CI coefficient (as in the second, 13th, and 14th excited states in Table 3), then $R_{\mathrm{e}}$ becomes shorter than


Figure 2. Charge distributions for atomic $\mathrm{La} 4 \mathrm{~s}, 4 \mathrm{p}_{-}, 4 \mathrm{~d}_{-}, 4 \mathrm{f}_{-}, 5 \mathrm{~s}$, and $5 \mathrm{p}_{-}$spinors. The charge distributions of $l_{-}$and $l_{+}$spinors for La$\left(5 p^{6}\right)^{3+}$ are close, so that we give $l_{-}$as representative. The spinors are those for $\mathrm{La}\left(5 p^{6}\right)^{3+}$ and the basis set for describing them is that of KTM. ${ }^{19}$


Figure 3. MC-QDPT potential curves for $\mathrm{LaF}^{+}$. The solid and the dashed lines respectively indicate observed and unobserved states.

TABLE 4: Spinor Energies (hartrees) and GAOPS of LaF ${ }^{+}$at $2.052 \AA$ with $\mathbf{C 4}^{a, b}$

${ }^{a}$ This is $R_{\mathrm{e}}$ of the LaF ground state from C4 MC-QDPT; the DFR total energy for $\mathrm{LaF}^{+}$is -8593.141760 hartree. ${ }^{b}$ The lower first to ninth spinors and the higher 31st to 145 th spinors are not shown to save space.
that of the ground state including tight 4 f - and 5 d -like spinors (see Figure 3 and Table 3). A dilute valence electron distribution leads to a stronger electrostatic interaction between the $\mathrm{La}^{2.5+}{ }_{-}$ $\left(5 p^{6} \mathrm{~d} * 0.4 \mathrm{f} * 0.1\right)$ and $\mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)$ ions and to a shorter $R_{\mathrm{e}}$. The calculated $\omega_{\mathrm{e}}$ values have some scope for improvement.

In concluding the $\mathrm{LaF}^{+}$calculations, we derived the correct spectroscopic constants, including the excitation energies $T_{0}$. In considering the f spectra, we should pay special attention to the orientation of the f spinors. Electron correlations between the valence and the N shell were significantly as well as correlations between the valence and the O shell.
3.2. LaF. 3.2.1. $L^{2} F^{+}$DFR Spinors for CAS and MC-QDPT Calculations on LaF. For LaF we used the model C4, where the frozen-core is $\mathrm{Zn}^{2+}(28)$ and $\mathrm{He}(2)$. We performed RFCA DFR calculations for $\mathrm{LaF}^{+}$, because $\mathrm{LaF}^{+}$provides an appropriate potential for the valence spinors for LaF , as $\mathrm{LaF}^{2+}$ does for $\mathrm{LaF}^{+}$. The results for $\mathrm{LaF}^{+}$are shown in Table 4, and the contour maps for the important spinors are displayed in Figure 4. The $\mathrm{LaF}^{+} 145 \mathrm{DFR}$ spinors are divided into the 17 active core, 30 valence, and 98 virtual spinors of LaF. Contrary to experiment, $\left(6 \mathrm{~s}_{+}\right)_{1 / 2}{ }^{1}$ becomes the ground state in DFR, rather than $(5 d)_{3 / 2}{ }^{1}$ (see the 18th and 26th spinors in Figure 4); a valence electron $\left\{(\mathrm{s})_{1 / 2} 0.8+(\mathrm{d})_{1 / 2} 0.2\right\}$ moves in the field generated by $\left\{\mathrm{La}^{2.6+}\left(5 \mathrm{p}^{6.0} \mathrm{~d} * 0.3 \mathrm{f} * 0.1\right) \mathrm{F}^{0.6-}\left(2 \mathrm{p}^{5.6}\right)\right\}^{2+}$ as indicated in Table 4. The attractive potential of $\mathrm{LaF}^{+}$in the DFR model is not strong enough to maintain the electron in the 5d shell, and correlation effects are needed to attach the $(5 \mathrm{~d})_{3 / 2}$ electron to the $\mathrm{LaF}^{2+}$ core.
3.2.2. Excited States with $T_{0} \leq 1.6 \mathrm{eV}$ and their Spectroscopic Constants. Using $30 \mathrm{LaF}^{+}$valence spinors for the respective partners of the Kramer's pair, we performed CASCI and MCQDPT calculations for LaF. Table 5 shows the assignments, excitation energies, and spectroscopic constants for states having $T_{0} \leq 1.6 \mathrm{eV}$. In the ground and single excited states having $\mathrm{c}_{1}|18 \underline{18}|$ or $\mathrm{c}_{1}|18 \underline{\mathrm{i}}|+\mathrm{c}_{2}|\underline{18 i}|$, many have weights $\left(\mathrm{C}_{1}{ }^{2}\right.$ or $\mathrm{C}_{1}{ }^{2}+$ $\mathrm{C}_{2}{ }^{2}$ ) $\geq 0.8$, indicating that use of the virtual spinors of $\mathrm{LaF}^{+}$as the valence spinors of the ground and excited states of LaF is
adequate. Potential curves for the states with $T_{0} \leq 1.6 \mathrm{eV}$ are shown in Figure 5.

Experimentally, the $T_{\mathrm{e}}$ values are well known but the $T_{0}$ values are not. The calculated $T_{0}$ values are close to the $T_{\mathrm{e}}$ values; the same may be true experimentally.

In the assignments, approximate GAOPs given by eq 2 and important configurations are included where the numbers in the determinant are the spinors in Table 6. We see no excited states with 4 f spinors having a single electron. The ground state of LaF is written symbolically as ( 6 s ), ${ }^{2}$ but the GAOPs show that it is $6 s^{1.4} 6 \mathrm{p}^{0.1} 5 \mathrm{~d}^{0.5}$. As in experiment, the symmetry $\Omega$ is calculated as $0^{+}$.

Within 0.3 eV above the ground state, three states are observed experimentally and are designated as $\left(6 s^{1} 5 d^{1}\right)^{3} \Delta$ with $\Omega=1,2$, and 3 . The present calculation reproduces these excitation energies with reasonable accuracy, and the GAOPs and $\Omega$ values support the experimental designation. We also confirm this assignment using the important CSFs in Table 5 and the shape of the d-mixed s-like 18th, pure d-like 26th and 27th spinors in Figure 4.

A small energy gap exists between the third and fourth excited states (see Table 5 and Figure 5). Experimentally, the fourth excited state is 0.39 eV from the third excited state; theoretically, it is 0.48 eV apart. The experimental and theoretical assignments are consistent.

The fifth to ninth excited states were designated experimentally as $\left(6 s^{1} 5 d^{1}\right){ }^{3} \Pi$ and ${ }^{1} \Pi$ with $\Omega=0^{-}, 0^{+}, 1,2$, and 1 . The present work shows that the $\Omega$ 's are correct but the experimental configurations are questionable, because the important CSFs and GAOPs in Table 5 suggest significant contributions from atomic p spinors; to see this, contrast the p-mixed d-like 19 and 29 spinors in the fifth to ninth excited states with the pure d-like 26 and 27 spinors in the first to fourth excited states (see Figure 4).

The 10th and 11th excited states are predicted theoretically to be around 1.46 eV above the ground state, but these are not found experimentally; the former is reported by Fahs and co-


Figure 4. Contour maps of densities of the valence spinors of the ground state of $\mathrm{LaF}^{+}$, together with spinor energies. The caption " $n$; pop ${ }_{1}\left(l_{ \pm}\right)_{\lambda / 2}$ $+\operatorname{pop}_{2}\left(l_{ \pm}^{\prime}\right)_{\lambda / 2} \epsilon^{\prime \prime}$, for the respective contour maps denotes the $n$th spinor having total electronic angular momentum $\lambda / 2$, contributed largely from the La $l_{ \pm}$and $l_{ \pm}^{\prime}$ spinors having electronic populations pop 1 and pop 2 and followed by the spinor energy in hartrees. The horizontal and vertical ( $z$ and $x$-) axes are in bohrs; $z$ covers -10 to 10 bohrs and $x 0$ to +10 bohrs. The circles on the $z$-axis at $z=0.0$ and 3.878 bohrs ( $2.052 \AA$ ) indicate the La and F nuclei, respectively. The outermost values of the contour line are $0.0001 e$ bohrs $^{-3}$. The contour maps for La 5 p and F 2 p are the sum of those for (the 12 th , the 13 th , and the 14 th ) and for (the 15 th, the 16 th, and the 17 th ) spinors in Table 4 . The value on a contour is twice that of the neighboring one outside it. The electron numbers inside the outermost line are between 0.996 and 1.001 , except for the La 5 p-like and F 2 p -like spinors for which the electron numbers are 3.000 and 3.001 respectively. (For the former, contributions from La 5 p are 2.877 and from others are 0.123 ; for the latter, contributions from F 2 p are 2.706 and from others are 0.295 .)

TABLE 5: Assignment of States, Excitation Energies, and Spectroscopic Constants from C4 MC-QDPT for LaF Having $\mathbf{T}_{0} \leq$ $1.6 \mathrm{eV}^{a}$

${ }^{a}$ TEs with and without zero-point vibrational energies are -8594.641557 and -8594.642772 hartrees, respectively. ${ }^{b}$ See ref 4 . ${ }^{c}$ The numbers in a determinant $|\ldots|$ are the spinors given in Table 4, and a spinor $(\underline{i})$ with an under bar indicates the Kramer's partner of $(i) .{ }^{d}$ See ref $15 .{ }^{e}$ See ref 9 . ${ }^{f}$ See ref $10 .{ }^{g}$ See ref 11 .


Figure 5. MC-QDPT potential curves for LaF ground and excited states with $T_{0} \leq 1.6 \mathrm{eV}$. The solid and the dashed lines, respectively, indicate observed and unobserved states. The 10th state is not given, because the state is predicted theoretically by Fahs and co-workers ${ }^{15}$ but no corresponding state is found by the present calculation.
workers, ${ }^{15}$ but is not shown in Figure 5. The 12th, 13th, and 15th excited states lie experimentally at $1.36,1.45$, and 1.55 eV above the ground state, respectively, and are designated as ${ }^{3} \Phi$ with $\Omega=2$, 3, and 4 , but the electronic configurations are not given. The calculated values are $1.63,1.73$, and 1.81 eV above the ground state, and the electron configuration suggested by GAOPs are $6 \mathrm{p}^{0.4} 5 \mathrm{~d}^{1.5} 4 \mathrm{f}^{0.1}, 6 \mathrm{p}^{0.4} 5 \mathrm{~d}^{1.6} 4 \mathrm{f}^{0.1}$, and $6 \mathrm{p}^{0.1} 5 \mathrm{~d}^{1.8}$ with $\Omega=2,3$, and 4 . The configurations given are reasonable, because the states are mainly spanned with determinants composed of (the p-mixed d-like 19 and pure d-like 26), (the p-mixed d-like 29/19 and pure d-like 26/27), and (the pure d-like 26 and pure d-like 27) spinors. The 14th state, found experimentally at 1.48 eV , is designated as ${ }^{1} \Sigma^{+}$with $\Omega=0^{+}$; this is a pure $(5 \mathrm{~d})^{2}$-like state having $71 \%(5 \mathrm{~d})_{3 / 2}{ }^{2}+18 \%(5 \mathrm{~d})_{5 / 2}{ }^{2}$.

We now discuss the spectroscopic constants. The calculated $R_{\mathrm{e}}$ for the ground state is $2.052 \AA$, and the experimental value is $2.023 \AA$. The experimental and calculated $R_{\mathrm{e}}$ values agree
well. We may find that as the states becomes higher, the 6 s GAOP decreases (see Table 5), and the excited state $R_{\mathrm{e}}$ increases (see Figure 5). The larger occupation in the diffuse 6 s (see Figure 4) enhances the interaction between the $\mathrm{La}^{2.6+}\left(5 \mathrm{p}^{6.0} \mathrm{~d}^{* 0.3}-\right.$ $\left.4 \mathrm{f}^{* 0.1}\right)$ and $\mathrm{F}^{0.6-}\left(2 \mathrm{p}^{5.6}\right)$ ionic cores, leading to smaller $R_{\mathrm{e}}$ values.

The calculated $\omega_{\mathrm{e}}$ values tend to decrease as the state becomes higher. Further investigation would be necessary to consider $\omega_{\mathrm{e}}$ for the ground state for which the calculated $\omega_{\mathrm{e}}$ is slightly smaller.

In conclusion, for the states with $T_{0} \leq 1.6 \mathrm{eV}$ we see good correspondence between the present calculations, LFT, ${ }^{15}$ and experiment, at least for $T_{0}$ and $T_{\mathrm{e}}$. However, the assigned electronic configurations show some discrepancy, as in the fifth to ninth excited states where $6 \mathrm{p}\left(\mathrm{p}^{*}\right)$ occupations are considerable.
3.2.3. Excited States with $1.6<T_{0} \leq 2.8 \mathrm{eV}$ and Their Spectroscopic Constants. Assignments and spectroscopic constants for states having $1.6<T_{0} \leq 2.8 \mathrm{eV}$ are shown in Table 6. Again, we have no excited states with 4 f spinors having a single electron occupation. The f-excited states may appear in the low-energy region, but only if the $n(>1)$ electron ground state includes f spinor(s).

We now discuss the 16th to 19th excited states. Three of these four are found experimentally to remain in the range, at 1.60, 1.61 , and 1.62 eV above the ground state. The experimental assignments are ${ }^{3} \Pi$ with $\Omega=0^{+}, 0$, and 2 . Our investigation indicates that the excitation energies for $0^{+}$and $0^{-}$are 1.68 and 1.63 eV , respectively. The $\Omega$ values are the same as experiment, and the calculated excitation energies are close to experiment. The experimental assignment suggests that the states at 1.60 and 1.61 eV have the same configuration because they both arise from ${ }^{3} \Pi$, but the present calculation suggests that the two states do not have the same configuration, as seen in Table 6; the lower is $(\mathrm{spf})^{0.6}(\mathrm{~d})^{1.5}$ and the higher is $(\mathrm{spf})^{1.3}(\mathrm{~d})^{0.7}$. We also cannot find the state 1.62 eV with $\Omega=2$; instead, we find the excited state with $\Omega=1$ at 1.63 eV above the ground state. This state might be observed if a further experiment is performed.

In the 20th to 30th states, the agreement between the experimental and calculational excitation energies is good except for the 28th excited state having $\Omega=4$; we doubt that MCQDPT works appropriately for this state. Contrary to experiment, the 26th and 28th states tend to have $5 \mathrm{~d}^{2}$ instead of the experimental configuration $5 \mathrm{~d}^{1} 6 \mathrm{p}^{1}$ (see the important CSF in

TABLE 6: Assignment of States, Excitation Energies, and Spectroscopic Constants from C4 MC-QDPT for LaF Having 1.6 < $T_{0} \leq 2.8 \mathrm{eV}^{a}$

${ }^{a}$ TEs with and without zero point vibrational energies are -8594.641557 and -8594.642772 hartrees, respectively. ${ }^{b}$ See ref $15 .{ }^{c}$ The numbers in a determinant $|\ldots|$ are the spinors given in Table 4 , and a spinor ( $i \underline{l}$ with an underbar indicates the Kramer's partner of $(i)$. ${ }^{d}$ See ref $4 .{ }^{e}$ See ref 5. ${ }^{f}$ See ref 7 . ${ }^{g}$ See ref 8 .

TABLE 7: GAOPs at Corresponding $R_{\mathrm{e}}$ and Spectroscopic Constants of $\mathrm{LaF}^{+}$and LaF calculated with MC-QDPT

| spinor | La s+ | La p | La d | Laf | F s+ | F p |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{LaF}^{+} \sum \mathrm{GAOP}_{i}(i=1,17)$ | 4.01 | 11.99 | 10.42 | 0.12 | 1.99 | 5.47 |
| $\begin{aligned} & \text { a-core: } \mathrm{La}^{2.5+}\left(\mathrm{d}^{*}\right)^{0.4}\left(\mathrm{f}^{*}\right)^{0.1} \mathrm{~F}^{0.5-}(2 \mathrm{p})^{5.5} \\ & \text { valence: } 5 \mathrm{~d}^{1} \\ & \text { spectroscopic constant }{ }^{a} \end{aligned}$ | 0.00 | $0.00$ | , 1.00 | $\begin{gathered} 0.00 \\ \mathrm{~S}_{\mathrm{e}} 5.86 \end{gathered}$ | 0.00 | 0.00 |
| LaF $\sum \mathrm{GAOP}_{i l}(i=1,17)$ | 4.01 | 12.00 | 10.34 | 0.11 | 1.99 | 5.56 |
| a-core: $\mathrm{La}^{2.6+}\left(\mathrm{d}^{*}\right)^{0.3}\left(\mathrm{f}^{*}\right)^{0.1} \mathrm{~F}^{0.6-}(2 \mathrm{p})^{5.6}$ valence $\mathrm{s}^{1.4} \mathrm{p}^{0.1} \mathrm{~d}^{0.5} \mathrm{f}^{0.0}$ spectroscopic constant ${ }^{c}$ | 1.38 | $0.11$ | $\begin{gathered} 0.48 \\ \omega_{\mathrm{e}} 534 \end{gathered}$ | $\begin{gathered} 0.02 \\ 5.72 \end{gathered}$ | 0.00 | 0.00 |

${ }^{a}$ The MC-QDPT dissociation energy is calculated using the $\mathrm{C}_{\infty v}$ double group: $\mathrm{TE}\left(\mathrm{LaF}^{+}: \Omega ; 3 / 2, \mathrm{R} ; 2.002 \AA\right)=-8594.434718, \mathrm{TE}\left(\mathrm{La}^{+}: \Omega ; 2\right)=$ -8494.527860 , and $\mathrm{TE}(\mathrm{F}: \Omega ; 1 / 2)=-99.691523$ hartrees. ${ }^{b}$ No experimental data are available. ${ }^{c}$ The MC-QDPT dissociation energy is calculated using the $\mathrm{C}_{\infty v}$ double group: $\mathrm{TE}\left(\mathrm{LaF}: \Omega ; 0^{+}, \mathrm{R} ; 2.052 \AA\right)=-8594.642772$, $\mathrm{TE}(\mathrm{La}: \Omega ; 3 / 2)=-8494.740356$, and $\mathrm{TE}(\mathrm{F}: \Omega ; 3 / 2)=-99.692041$ hartrees . ${ }^{d}$ Experimental value cited in ref 14 .

Table 6; for example, the 26th state is composed of $\mid \mathrm{p}$-mixed 20th, pure d-like 27th| and |p-mixed 19th, pure d-like 27th|).

The calculated $\omega_{\mathrm{e}} \mathrm{s}$ and the experimental values do not agree very well, and warrant further investigation of the potential curves for the higher states.

So far we have discussed mainly the excited states of $\mathrm{LaF}^{+}$ and LaF . We finally discuss the characteristics of the chemical bond in the ground state of $\mathrm{LaF}^{+}$and LaF .
3.3. Chemical Bond in $\mathrm{LaF}^{+}$and LaF. Table 7 summarizes the MC-QDPT GAOPs and spectroscopic constants for the ground states of $\mathrm{LaF}^{+}$and LaF . The ion cores for $\mathrm{LaF}^{+}$and LaF are almost $\left\{\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d}^{* 0.4} \mathrm{f}^{* 0.1}\right) \mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$. The valence electrons are almost entirely located at $\mathrm{La}^{2.5+}$ in both $\mathrm{LaF}^{+}$and LaF.

For the $\mathrm{LaF}^{+}$ground state ${ }^{2} \Delta(\Omega=3 / 2)$, the valence electron is in the atomic-like La 5 d spinor. The attraction potential in $\mathrm{La}^{2.5+} \mathrm{F}^{0.5-}$ is not strong enough to hold the 4 f electron but not so weak as to hold the 6 s electron; recall that if the nuclear attraction is very strong compared to the electron-electron repulsion, the $4 \mathrm{~d}_{5 / 2}$ and $4 f_{5 / 2}$ are degenerate, and in this case the electron is naturally captured in 4 f .

For the LaF ground state $\mathrm{X}^{1} \Sigma^{+}\left(\Omega=0^{+}\right)$, two electrons are located at the $\mathrm{La}^{2.5+}$ having strong s-d hybridizations. The attraction potential in the ground state given by $\left(\mathrm{La}^{2.5+} \mathrm{F}^{0.5-}\right)^{2+}$ is not strong enough to hold two 5 d -like electrons as the $\mathrm{La}^{3+}{ }^{3+}$ $\left(5 p^{6}\right)$ core in the gaseous $\mathrm{La}^{+}$ion and is also not strong enough to keep one electron in the 6 s and the other in the 5 d spinor (we find many low lying ( $6 s^{1} 5 \mathrm{~d}^{1}$ ) states instead, as was discussed).

We conclude that the chemical bond in $\mathrm{LaF}^{+}$and LaF is constructed through a formation of $\left\{\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d}^{* 0.4} \mathrm{f} * 0.1\right)\right.$ -$\left.\mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$, which arises from the interaction between $\mathrm{La}^{2+}\left(5 \mathrm{p}^{6} 5 \mathrm{~d}^{1}\right)$ and $\mathrm{F}\left(2 \mathrm{p}^{5}\right)$ (see the F 2 p - and La 5 p-like spinors in Figures 1 and 4), forcing valence electron(s) to move around $\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d} * 0.4 \mathrm{f} * 0.1\right)$. In that case, the spectroscopic constants, $R_{\mathrm{e}}$ and $\omega_{\mathrm{e}}$, should be similar for $\mathrm{LaF}^{+}$and LaF , as should be the dissociation energies $D_{\mathrm{e}}$. The spectroscopic constants in Table 7 confirm this; the values of $D_{\mathrm{e}}$ for $\mathrm{LaF}^{+}$and LaF are respectively 5.86 and 5.74 eV . The augmented electron in $\mathrm{LaF}^{+}$, which is located at La , is responsible for the reduction in the total energy, giving the positive electron affinity ( 5.66 eV ) for $\mathrm{LaF}^{+}$with only a small change in the molecular shape. A similar result was obtained by Franzreb and co-workers, using a nonrelativistic density functional approach. ${ }^{35}$

We believe that the formal charges for +1 and -1 for the La and F ions are reasonable, because the electrons in $\mathrm{F}^{-}$flow partly back to La through $\mathrm{d}^{*}$ and $\mathrm{f}^{*}$, and by treating the GAOPs for these orbitals as $\mathrm{F}^{-}$the charges on F become -0.99 (GAOPs in Table 6).

Finally, we discuss the validity of the LFT calculations. The present calculation shows why the valence electron(s) localize(s) at La. We can therefore discuss the electronic structure of the molecules by considering only the valence electrons, provided one can properly account for the correlation effects between the valence electrons and the $\mathrm{La}^{2.5+} \mathrm{F}^{0.5-}$ ion cores in various empirical parameters. Our discussion gives support to the LFT calculations.

## 4. Conclusion

We have used the RFCA four-component DFR method, CASCI, and MC-QDPT to study the electronic structures of the $\mathrm{LaF}^{+}$and LaF molecules.

The calculated excitation energies for $\mathrm{LaF}^{+}$agree with the experimental values; the errors are within 0.4 eV . For example, the highest observed state ${ }^{2} \Pi$ is 3.77 eV above the ground state, and the calculated ${ }^{2} \Pi$ is 4.09 eV above. The calculated excitation energies also agree with those of LFT, indicating that the empirical parameters in LFT implicitly include correlation effects between the valence and core electrons. When considering $f$ spectra, we paid attention to the orientation of the $f$ spinors.

For LaF, 30 states having excitation energy $\leq 2.80 \mathrm{eV}$ were discussed. The excitation energies (and the angular momentum $\Omega$ ) given by experiment and calculation agree to within 0.4 eV except for a single anomaly, but in some cases the electron configurations do not. In such cases, we have confidence in our assignment, especially in the lower excited states. The spectroscopic constants have also been considered.

The characteristics of the chemical bonds in $\mathrm{LaF}^{+}$and LaF have been discussed in detail. Common features in the ground state of $\mathrm{LaF}^{+}$and LaF are the following: (1) interaction between $\mathrm{La}^{2+}(5 \mathrm{p})^{6}(5 \mathrm{~d})^{1}$ and $\mathrm{F}(2 \mathrm{p})^{5}$ determines the electronic structure framework of the two molecules, forming $\left\{\mathrm{La}^{2.5+}\left(5 \mathrm{p}^{6} \mathrm{~d} * 0.4 \mathrm{f} * 0.1\right)\right.$ -$\left.\mathrm{F}^{0.5-}\left(2 \mathrm{p}^{5.5}\right)\right\}^{2+}$; and (2) the valence electron(s) are almost completely localized at the La atom, because the positive ionic core of La attracts the electrons and the negative charges of $\mathrm{F}^{-}$ repel the additional electrons in the region of $\mathrm{F}^{-}$. This second point validates the LFT calculations.

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